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Minimal Flavour Violation and Anomalous Top Decays

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Top quark physics at the LHC may open a window to physics beyond the standard model and even lead us to an understanding of the phenomenon "flavour". However, current flavour data is a strong hint that no "new physics" with a generic flavour structure can be expected in the TeV scale. In turn, if there is "new physics" at the TeV scale, it must be "minimally flavour violating". This has become a widely accepted assumption for "new physics" models. In this paper we propose a way to test the concept of minimal flavour violation for the anomalous charged Wtq, $q \in \{d, s, b\}$, and flavour-changing Vtq, $q \in \{u, c\}$ and $V \in \{Z, \gamma, g\}$, couplings within an effective field theory framework, i.e. in a model independent way. We perform a spurion analysis of our effective field theory approach and calculate the decay rates for the anomalous top-quark decays in terms of the effective couplings for different helicities by using a two-Higgs doublet model of type II (2HDM-II), under the assumption that the top-quark is produced at a high energy collision and decays as a quasi-free particle.

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I. INTRODUCTION

Due to its large mass, $m_t = 173.2 \pm 0.9$ GeV [1], the top-quark seems to play a special role, therefore top-quark physics may give hints to physics beyond the standard model (SM). Assuming that the Higgs mechanism is indeed the way particles obtain their masses, the top-quark is the only quark with a Yukawa coupling of a "natural" size, i.e. a coupling of order unity. Thus it may in particular lead us to an understanding of the phenomenon "flavour", which is in the SM encoded in the Yukawa couplings.

Given the fact that up to now we do not have any hint at a specific model for "new physics" (NP), and that the LHC [2] will produce a large amount of top-quarks [3, 4], it is be desirable to have an approach which is as model independent as possible to analyse the data. Hence we will not pick a specific model here, rather we shall refer to an effective theory description of possible NP. This approach is well known and we shall gather the necessary relations in the next section.

However, including up to dimension-6 operators in this approach, a large number of unknown couplings appears, which are a priori unconstrained. From present data we may obtain limits on certain couplings which have to be included in the analysis. In particular, flavour physics rules out a generic flavour structure for the dimension-6 operators, and the flavour constraints can be readily incorporated by the assumption of minimal flavour violation (MFV).

MFV has become a popular assumption to avoid flavour constraints in many new physics models. On the other hand, it is important to test, if a hint at NP is compatible with MFV or not. Hence it is desirable to have the possibility to test the MFV hypothesis without recurrence to a specific new physics model, in which case one can only make use of the effective theory approach.

There is already a large number of analyses of anomalous top couplings in the literature, some recent work can be found in [5–8]. However, these papers either deal with flavour-diagonal anomalous couplings or do not take into account the constraints from MFV. In the present paper we propose a way to perform a test of the MFV hypothesis in anomalous flavour-changing top couplings. The basic idea is to make use of the MFV constraints on the flavour structure of the $t \to qV$ couplings, where q is a quark an V a gauge boson. However, this is still not restrictive enough to allow for a simple analysis and hence we will be forced to make additional assumptions which we shall keep as simple as possible.

The paper is organised as follows: in the next section we collect the relevant dimension-6 operators for an effective description at the top-mass scale. In section III we give the MFV relations between the coupling constants of these operators and project out the relevant operators for charged and neutral current top decays. In section IV we compute the rates for top decays into lighter quarks under the emission of gluons, photons and weak bosons and derive relations between different decay rates which may serve as a test of MFV and conclude in section V.

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II. EFFECTIVE APPROACH WITH TWO HIGGS DOUBLETS

We consider the SM as the dimension-4 part of an effective theory; hence physics at a large scale Λ beyond the SM manifests itself through the presence of higher dimensional operators, suppressed by powers of the scale Λ. This is generically true for any new physics model with degrees of freedom at scales Λ. However, all particles constituting the SM have been found, however, the symmetry-breaking sector is not yet fixed, although the recent discovery at the LHC is very likely a Higgs particle [9, 10].

In the present paper we shall take this into account by assuming a two-Higgs doublet model of type II (2HDM-II), which allows an easy contact to supersymmetric models (SUSY), to the SM with a single Higgs doublet and also to heavy Higgs models using non-linear representations [11–13]. Focussing on quarks only and using the notation

$$Q_L = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right) \ , \tag{1}$$

$$u_R = (u_R, c_R, t_R) \, , d_R = (d_R, s_R, b_R) \ , \tag{2}$$

the Yukawa couplings of the 2HDM-II model can be written in terms of two Higgs doublet fields Φ_1 and Φ_2 with hypercharge $Y = 1$ and otherwise identical quantum numbers, as

$$-\mathcal{L}^{\rm 2HDM}_{\rm Yuk} = \bar{Q}_L Y_D \Phi_1 d_R + \bar{Q}_L Y_U \tilde{\Phi}_2 u_R + \text{h.c.} \ , \tag{3}$$

where $\tilde{\Phi}$ is the charge conjugated Higgs doublet which is given by

$$\tilde{\Phi} = i\tau_2 \Phi^* \ . \tag{4}$$

"New physics" beyond this SM-like dimension-4 piece is parametrized in terms of higher dimensional operators [14–16]

$$\mathcal{L} = \mathcal{L}_{4D} + \frac{1}{\Lambda}\mathcal{L}_{5D} + \frac{1}{\Lambda^2}\mathcal{L}_{6D} + \dots \ , \tag{5}$$

where $\mathcal{L}_{4D} \equiv \mathcal{L}_{\rm SM}$, and the new contributions $\mathcal{L}_{5D}$, $\mathcal{L}_{6D}$, ..., have to be symmetric under the SM gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. It turns out, that for quarks there is no dimension five operator compatible with this symmetry and thus the next-to-leading terms in the Λ^{-1} expansion are of dimension six or higher. The number of possible operators is already for a single Higgs boson quite large [16, 17].

We are going to consider this effective Lagrangian at the top-quark mass scale, which we identify with the electroweak scale $\mu \sim m_t$. Furthermore, we are interested in processes leading to anomalous, flavour-changing couplings of the top-quark to the gauge bosons, and hence it is sufficient for us to look at operators which are bilinear in the quark fields. It is useful to classify the operators according to the helicities of the quark fields, left-left (LL), right-right (RR) and left-right (LR). Using this notation, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\rm 2HDM} + \frac{1}{\Lambda^2}\sum_i \mathcal{C}^{(i)}_{LL}\mathcal{O}^{(i)}_{LL} + \frac{1}{\Lambda^2}\sum_i \mathcal{C}^{(i)}_{RR}\mathcal{O}^{(i)}_{RR} + \frac{1}{\Lambda^2}\sum_i \mathcal{C}^{(i)}_{LR}\mathcal{O}^{(i)}_{LR} + \dots \ , \tag{6}$$

where the $\mathcal{C}^{(i)}_{hh'}$ are generic coupling constants which can in principle be calculated in specific models of new physics.

It is an easy exercise to write down all possible operators of dimension-6, which are bilinear in the quark fields [16]. However, not all of them are independent when applying the equations of motion. Furthermore, within the 2HDM-II model, "Flavour-Changing Neutral Currents" (FCNCs) and large CP violation are naturally suppressed by the imposed discrete $\mathcal{Z}_2$ symmetry, forbidding $\Phi_1 \leftrightarrow \Phi_2$ transistions [18]. Hence the set of independet operators for purely left-handed transitions can be choosen as [16]:

$$\begin{aligned}
\mathcal{O}^{ij(3)}_{LL} &= \left(\Phi_1^\dagger i D_\mu \Phi_1\right)\left(\bar{Q}_{Li}\gamma^\mu Q_{Lj}\right) , \\
\mathcal{O}^{ij(4)}_{LL} &= \left(\Phi_2^\dagger i D_\mu \Phi_2\right)\left(\bar{Q}_{Li}\gamma^\mu Q_{Lj}\right) , \\
\mathcal{O}^{ij(5)}_{LL} &= \left(\Phi_1^\dagger \tau_I i D_\mu \Phi_1\right)\left(\bar{Q}_{Li}\tau_I\gamma^\mu Q_{Lj}\right) , \\
\mathcal{O}^{ij(6)}_{LL} &= \left(\Phi_2^\dagger \tau_I i D_\mu \Phi_2\right)\left(\bar{Q}_{Li}\tau_I\gamma^\mu Q_{Lj}\right) ,
\end{aligned} \tag{7}$$

and for purely right-handed transitions we have

$$\begin{aligned}
\mathcal{O}^{ij(2)}_{RR} &= \left(\Phi_2^\dagger i D_\mu \Phi_2\right)\left(\bar{u}_{Ri}\gamma^\mu u_{Rj}\right) , \\
\mathcal{O}'^{ij(2)}_{RR} &= \left(\Phi_1^\dagger i D_\mu \Phi_1\right)\left(\bar{d}_{Ri}\gamma^\mu d_{Rj}\right) , \\
\mathcal{O}^{ij(3)}_{RR} &= \left(\tilde{\Phi_2}^\dagger i D_\mu \Phi_1\right)\left(\bar{u}_{Ri}\gamma^\mu d_{Rj}\right) ,
\end{aligned} \tag{8}$$

where D_μ denotes the covariant derivative of the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry. For the transitions from left to right-handed helicities we have

$$\begin{aligned}
\mathcal{O}^{ij(4)}_{LR} &= \left(\bar{Q}_{Li}\sigma^{\mu\nu}\tau_I u_{Rj}\right)\tilde{\Phi}_2 W^I_{\mu\nu} + \text{h.c.}, \\
\mathcal{O}^{ij(5)}_{LR} &= \left(\bar{Q}_{Li}\sigma^{\mu\nu} u_{Rj}\right)\tilde{\Phi}_2 B_{\mu\nu} + \text{h.c.}, \\
\mathcal{O}'^{ij(4)}_{LR} &= \left(\bar{Q}_{Li}\sigma^{\mu\nu}\tau_I d_{Rj}\right)\Phi_1 W^I_{\mu\nu} + \text{h.c.}, \\
\mathcal{O}'^{ij(5)}_{LR} &= \left(\bar{Q}_{Li}\sigma^{\mu\nu} d_{Rj}\right)\Phi_1 B_{\mu\nu} + \text{h.c.} \ ,
\end{aligned} \tag{9}$$

where $W^{I=1,2,3}_{\mu\nu}$ and $B_{\mu\nu}$ denote the field strength of $SU(2)_W$ and $U(1)_Y$ symmetries, respectively, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

In addition to these operators leading to anomalous weak couplings we also can have anomalous coupling to gluons which read

$$\begin{aligned}
P^{ij(5)}_{LR} &= \left(\bar{Q}_{Li}\sigma^{\mu\nu}T^a u_{Rj}\right)\tilde{\Phi}_2 G^a_{\mu\nu} + \text{h.c.}, \\
P'^{ij(5)}_{LR} &= \left(\bar{Q}_{Li}\sigma^{\mu\nu}T^a d_{Rj}\right)\Phi_1 G^a_{\mu\nu} + \text{h.c.} \ ,
\end{aligned} \tag{10}$$

where T^a are the generators of $SU(3)_C$ and $G^a_{\mu\nu}$ is the gluon field strength. Note that all these operators carry flavour indices i, j and hence the coupling constants in Eq. (6) are actually 3×3 matrices in flavour space. Thus it is evident that generic parametrization is pretty much useless due to the large number of unknown parameters.

III. MINIMAL FLAVOUR VIOLATION

Data on flavour processes restricts the possible couplings in Eq. (6) severely. Since currently there is no indication from flavour processes of new effects at the TeV scale, any NP at that scale must be "minimally flavour violating" [19, 20], i.e. the new physics couplings obey the same flavour-suppression pattern as the standard model processes.

The most economical way to implement this idea has been advocated in [21], where a

$$\mathcal{G}_F = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \tag{11}$$

flavour symmetry has been introduced, which is (up to for our purposes irrelevant $U(1)$ factors) the largest flavour symmetry which is compatible with the SM. Under this symmetry the quarks transform according to

$$Q_L \sim (3,1,1)\ , \tag{12}$$

$$u_R \sim (1,3,1)\ ,\ d_R \sim (1,1,3)\ , \tag{13}$$

while the SM gauge and the Higgs fields are singlets with respect to $\mathcal{G}_F$.

In the SM, this symmetry is broken only by the Yukawa couplings Y_U and Y_D shown in Eq. (3). Following Ref. [21] these Yukawa couplings can be introduced as spurion fields with the transformation property

$$Y_U \sim (3,\bar{3},1)\ ,\ Y_D \sim (3,1,\bar{3})\ , \tag{14}$$

such that the Yukawa interaction (3) is rendered invariant. "Freezing" the spurion fields to the actual values of the Yukawa couplings yields the $\mathcal{G}_F$ symmetry breaking in the SM.

This spurion analysis can be extended to any new physics model as well as to our effective field theory approach. To this end we insert the minimum number of spurions into the set of higher dimensional operators, which are required to be Lorentz and gauge invariant as well as flavour invariant. Thus we get, omitting some trivial structures, which lead to flavour violation

$$\sum_{i,j} \mathcal{C}^{ij}_{LL} \left(\bar{Q}_{Li} \cdots Q_{Lj}\right) = \bar{Q}_L \left[\alpha_{LL} \mathbb{1} + \beta_{LL}\, Y_U Y_U^\dagger + \eta_{LL}\ Y_D Y_D^\dagger\right] \cdots Q_L\ , \tag{15}$$

$$\sum_{i,j} \mathcal{C}^{ij}_{RR} \left(\bar{u}_{Ri} \cdots u_{Rj}\right) = \alpha_{RR} \left(\bar{u}_R \left[Y_U^\dagger Y_D Y_D^\dagger Y_U\right] \cdots u_R\right)\ , \tag{16}$$

$$\sum_{i,j} \mathcal{C}'^{ij}_{RR} \left(\bar{d}_{Ri} \cdots d_{Rj}\right) = \beta_{RR} \left(\bar{d}_R \left[Y_D^\dagger Y_U Y_U^\dagger Y_D\right] \cdots d_R\right)\ , \tag{17}$$

$$\sum_{i,j} \mathcal{C}''^{ij}_{RR} \left(\bar{d}_{Ri} \cdots u_{Rj}\right) = \eta_{RR} \left(\bar{d}_R \left[Y_D^\dagger Y_U\right] \cdots u_R\right)\ , \tag{18}$$

$$\sum_{i,j} \mathcal{C}^{ij}_{LR} \left(\bar{Q}_{Li} \cdots u_{Rj}\right) = \bar{Q}_L \left[\lambda_U Y_U + \alpha_{LR} Y_D Y_D^\dagger Y_U\right] \cdots u_R\ , \tag{19}$$

$$\sum_{i,j} \mathcal{C}'^{ij}_{LR} \left(\bar{Q}_{Li} \cdots d_{Rj}\right) = \bar{Q}_L \left[\lambda_D Y_D + \beta_{LR} Y_U Y_U^\dagger Y_D\right] \cdots d_R\ , \tag{20}$$

where the ellipses denote the Dirac, colour and weak $SU(2)$ matrices that appear in the operators.

The coefficients $\alpha_{LL} \cdots \beta_{LR}$ are expected to have a "natural" size. The precise meaning of this statement depends on the way the NP effects enter the stage. A tree-level induced NP effect (e.g. by a tree-level exchange of a new particle with mass Λ) will induce coefficients $\alpha_{LL} \cdots \beta_{LR} \sim \mathcal{O}(1)$, while loop-induced NP effects will suffer from the typical loop-suppression factor $1/(16\pi^2)$ and hence we would have $\alpha_{LL} \cdots \beta_{LR} \sim \mathcal{O}(10^{-2})$.

The physical quark fields are the mass eigenstates, which are defined in such a way that the neutral component of the terms proportional to λ_U and λ_D in (19) and (20) is diagonal, since this contribution is exactly of the form of the mass terms in the SM Lagrangian. This is achieved by picking a basis of the Y_U and Y_D where

$$Y_U = Y_U^{\mathrm{diag}}\ , \quad Y_D = V_{\mathrm{CKM}} Y_D^{\mathrm{diag}}\ , \tag{21}$$

where V_{CKM} is the CKM rotation from the weak to the mass eigenbasis: $d_L^{\mathrm{weak}} = V_{\mathrm{CKM}} d_L^{\mathrm{mass}}$.

Resolving the terms in Eqs. (15)–(20) into charged an neutral components, one finds in terms of mass eigenstates

for the charged component from (15) (from here all quark fields are mass eigenstates)

$$\sum_{i,j} \mathcal{C}_{LL}^{ij} \left(\bar{Q}_{Li} \cdots \tau_+ Q_{Lj}\right) = \bar{u}_L \cdots \left[\alpha_{LL} \mathbb{1} + \beta_{LL} \left(Y_U^{\rm diag}\right)^2 + \eta_{LL} \left(Y_D^{\rm diag}\right)^2\right] V_{\rm CKM} d_L \ , \tag{22}$$

with $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$, while we get for the neutral components

$$\frac{1}{2} \sum_{i,j} \mathcal{C}_{LL}^{ij} \left(\bar{Q}_{Li} \cdots (1+\tau_3) Q_{Lj}\right) = \eta_{LL} \bar{u}_L \cdots V_{\rm CKM} \left(Y_D^{\rm diag}\right)^2 V_{\rm CKM}^\dagger u_L \ , \tag{23}$$

$$\frac{1}{2} \sum_{i,j} \mathcal{C}_{LL}^{ij} \left(\bar{Q}_{Li} \cdots (1-\tau_3) Q_{Lj}\right) = \eta_{LL} \bar{d}_L \cdots V_{\rm CKM}^\dagger \left(Y_U^{\rm diag}\right)^2 V_{\rm CKM} d_L \ . \tag{24}$$

For the remaining helicity combinations we get

$$\sum_{i,j} \mathcal{C}_{RR}^{ij} \left(\bar{u}_{Ri} \cdots u_{Rj}\right) = \alpha_{RR} \left(\bar{u}_R \left[Y_U^{\rm diag} V_{\rm CKM} \left(Y_D^{\rm diag}\right)^2 V_{\rm CKM}^\dagger Y_U^{\rm diag}\right] \cdots u_R\right) \ , \tag{25}$$

$$\sum_{i,j} \mathcal{C}_{RR}^{\prime ij} \left(\bar{d}_{Ri} \cdots d_{Rj}\right) = \beta_{RR} \left(\bar{d}_R \left[Y_D^{\rm diag} V_{\rm CKM}^\dagger \left(Y_U^{\rm diag}\right)^2 V_{\rm CKM} Y_D^{\rm diag}\right] \cdots d_R\right) \ , \tag{26}$$

$$\sum_{i,j} \mathcal{C}_{RR}^{\prime\prime ij} \left(\bar{d}_{Ri} \cdots u_{Rj}\right) = \eta_{RR} \left(\bar{d}_R \left[Y_D^{\rm diag} V_{\rm CKM}^\dagger Y_U^{\rm diag}\right] \cdots u_R\right) \ . \tag{27}$$

Finally, Eqs. (19) and (20) have to be split into charged and neutral components. Omitting flavour diagonal contributions, we get

$$\left.\sum_{i,j} \mathcal{C}_{LR}^{ij} \left(\bar{Q}_{Li} \cdots u_{Rj}\right)\right|_{\rm charged} = \bar{d}_L \left[\lambda_U V_{\rm CKM}^\dagger Y_U^{\rm diag} + \alpha_{LR} \left(Y_D^{\rm diag}\right)^2 V_{\rm CKM}^\dagger Y_U^{\rm diag}\right] \cdots u_R \ , \tag{28}$$

$$\left.\sum_{i,j} \mathcal{C}_{LR}^{ij} \left(\bar{Q}_{Li} \cdots u_{Rj}\right)\right|_{\rm neutral} = \alpha_{LR} \bar{u}_L \left[V_{\rm CKM} \left(Y_D^{\rm diag}\right)^2 V_{\rm CKM}^\dagger Y_U^{\rm diag}\right] \cdots u_R \ , \tag{29}$$

$$\left.\sum_{i,j} \mathcal{C}_{LR}^{\prime ij} \left(\bar{Q}_{Li} \cdots d_{Rj}\right)\right|_{\rm charged} = \bar{u}_L \left[\lambda_D V_{\rm CKM} Y_D^{\rm diag} + \beta_{LR} \left(Y_U^{\rm diag}\right)^2 V_{\rm CKM} Y_D^{\rm diag}\right] \cdots d_R \ , \tag{30}$$

$$\left.\sum_{i,j} \mathcal{C}_{LR}^{\prime ij} \left(\bar{Q}_{Li} \cdots d_{Rj}\right)\right|_{\rm neutral} = \beta_{LR} \bar{d}_L \left[V_{\rm CKM}^\dagger \left(Y_U^{\rm diag}\right)^2 V_{\rm CKM} Y_D^{\rm diag}\right] \cdots d_R \, . \tag{31}$$

Thus the flavour structure of the operators can be fixed by the assumption of MFV, and hence the number of independent couplings is reduced to the number of operator structures listed in Sec. II.

Note that the entries in $Y_U^{\rm diag}$ and $Y_D^{\rm diag}$ are small except for the one entry in $Y_U^{\rm diag}$, corresponding to the top mass. Also, for large $\tan\beta$ also $Y_D^{\rm diag}$ may contain a large entry related to the bottom mass. It has been pointed out in Ref. [22] that this may spoil the expansion in powers of the spurion insertions. We will not go into any details here and restrict our analysis to the minimum number of spurion insertions.

In most of the analyses using effective theory approaches, unknown couplings are treated "one at a time", which means that one coupling is varied with all other couplings set to zero. In this paper we shall propose a slightly different scheme, which automatically implements the relations among the couplings implied by MFV. We will either set all the couplings to be unity, $\alpha_{LL} \cdots \beta_{LR} \equiv 1$ ("tree-induced scenario"), and vary the scale Λ, or we set $\alpha_{LL} \cdots \beta_{LR} \equiv 1/(16\pi^2)$ ("loop-induced scenario"), and vary the scale Λ. Alternatively, we may fix the scale Λ (e.g. at 1 TeV), which means that we identify all the couplings $\alpha_{LL} \cdots \beta_{LR}$ and study the constraints on the remaining single parameter.

The rationale behind this idea is that in a truely minimally flavour violating scenario all the remaning couplings should be natural. In turn this means that - up to the hierarchies implied by MFV - no further hierarchical structures should emerge. Without going into the

details of a specific NP model, there is no way to infer the detailed couplings; if we want to stick to a model independent approach there is no alternative to such a crude scheme.

IV. DECAY RATES OF TOP QUARKS

In the remainder of the paper we shall focus on processes with top-quarks and their anomalous couplings to gauge bosons. As mentioned in the previous section we use an effective theory approach to study anomalous, flavour-changing top couplings at the weak scale $\mu \sim m_t$. The MFV hypothesis allows us to predict relative sizes of couplings for different flavours in the final state; in turn, this may be used as a test of MFV in top decays, once anomalous decays have been discovered.

A. Charged Currents

The first class of decays are the charged currents from couplings of the form Wtq, $q \in \{d, s, b\}$. Taking into account the various helicity combinations, the effective interaction for the charged current couplings has the general form

$$\mathcal{L}_{\rm eff} = \sum_{q=d,s,b} \frac{g_2}{\sqrt{2}} \Big\{ -\bar{q}\gamma^\mu \left(L_1^q P_L + R_1^q P_R\right) t W_\mu^- \\ -(i\partial)_\nu \left[\bar{q}\frac{i\sigma^{\mu\nu}}{M_W}\left(L_2^q P_L + R_2^q P_R\right) t\right] W_\mu^- \Big\}, \quad (32)$$

where $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ denote the chiral projectors. Aplying the MFV hypothesis we get for the couplings

$$\begin{aligned} L_1^q =& V_{tq}^* \left[1 + \frac{\alpha_{LL}^{(5)}}{2}\frac{v_1^2}{\Lambda^2} + \frac{\alpha_{LL}^{(6)}}{2}\frac{v_2^2}{\Lambda^2}\right] = V_{tq}^* + \delta L_1^q \ , \\ R_1^q =& V_{tq}^* \eta_{RR}^{(3)} \frac{m_q m_t}{\Lambda^2} \ , \\ L_2^q =& 2V_{tq}^* \lambda_D^* \frac{m_q v}{\Lambda^2} \ , \\ R_2^q =& 2V_{tq}^* \lambda_U \frac{m_t v}{\Lambda^2} \ , \end{aligned} \quad (33)$$

where $v_1 \equiv v\cos\beta$ and $v_2 \equiv v\sin\beta$ are the vacuum expectation values of the Higgs fields Φ_1 and Φ_2, respectively. Note that we have kept the SM contribution in L_1^q and defined δL_1^q to be the possible NP piece. Furthermore, the parameter $v^2 = v_1^2 + v_2^2$ is fixed by the W-boson mass, $M_W^2 = \frac{1}{4}g_2^2 v^2$, where g_2 is the weak $SU(2)$ coupling constant, or equivalently by the Fermi constant, $G_F = 1/(\sqrt{2}v^2)$.

As discussed above, the remaining unknown couplings in Eqs. (33) are generically of order unity, and hence in MFV we have the order-of-magnitude estimate

$$\begin{aligned} R_1^q \sim \frac{2m_q m_t}{v^2}\delta L_1^q \ , \ L_2^q \sim \frac{4m_q}{v}\delta L_1^q \ , \\ R_2^q \sim \frac{4m_t}{v}\delta L_1^q \ . \end{aligned} \quad (34)$$

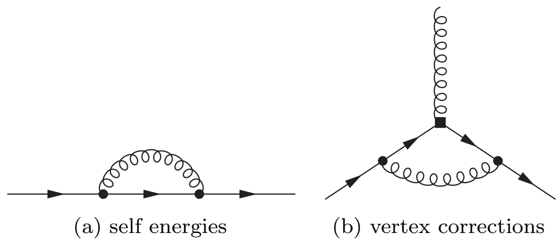

Figure 1. Feynman diagrams for the calculation of the anomalous dimension for the charged electroweak case.

Although the renormalization group flow is not expected to change the orders of magnitude, it is still important to know the renormalization effects for a quantitative analysis. We expect the relations (34) to hold at the high scale $\Lambda \gg \mu$, and so we have to scale down to the scale of the top mass, $\mu \sim m_t$, where the measurement takes place.

We focus on QCD effects only and consider the diagrams shown in Figs. 1(a) and 1(b). The left and right handed current do not have an anomalous dimension, and hence

$$L_1^q(m_t) = L_1^q(\Lambda) \ , \quad R_1^q(m_t) = R_1^q(\Lambda) \ . \quad (35)$$

The helicity-changing contributions have an anomalous dimension which has to be equal for both helicity combinations. To leading order one finds

$$\gamma^T(\alpha_s) = \frac{2\alpha_s}{3\pi} \ , \quad (36)$$

which yields for the running from Λ to $\mu \sim m_t$ for the two remaining couplings

$$\begin{aligned} L_2^q(m_t) = L_2^q(\Lambda)\left(\frac{\alpha_s(\Lambda)}{\alpha_s(m_t)}\right)^{\frac{4}{3\beta_0}} \ , \\ R_2^q(m_t) = R_2^q(\Lambda)\left(\frac{\alpha_s(\Lambda)}{\alpha_s(m_t)}\right)^{\frac{4}{3\beta_0}} \ , \end{aligned} \quad (37)$$

with

$$\beta_0 = \frac{11n_c - 2n_f}{3} \ , \quad (38)$$

where n_c and n_f are the numbers of colours and quark flavours, respectively.

From the effective operators we may calculate the amplitudes for top decays, taking into account a possible NP effect. To this end, we use Eq. (32) and compute the decay rates for the decay of an unpolarized top-quark into a down type quark q and an on-shell W-boson. The analysis of the W decay products allows us to reconstruct its polarization, which is either longitudinal, left- or right-handed. The corresponding rates read [23, 24]

$$\Gamma(t \to qW_0) = \frac{g_2^2|\vec{q}|}{32\pi}\left\{\frac{m_t^2}{M_W^2}\left[|L_1^q|^2+|R_1^q|^2\right]\left(1-x_W^2-2x_q^2-x_W^2x_q^2+x_q^4\right)-4x_q\mathrm{Re}\left\{L_1^qR_1^{q*}\right\}\right.$$
$$+\left[|L_2^q|^2+|R_2^q|^2\right]\left(1-x_W^2+x_q^2\right)-4x_q\mathrm{Re}\left\{L_2^qR_2^{q*}\right\}$$
$$-2\frac{m_t}{M_W}\mathrm{Re}\left(L_1^qR_2^{q*}+L_2^qR_1^{q*}\right)\left(1-x_W^2-x_q^2\right)$$
$$\left.+2\frac{m_t}{M_W}x_q\mathrm{Re}\left\{L_1^qL_2^{q*}+R_2^qR_1^{q*}\right\}\left(1+x_W^2-x_q^2\right)\right\} \tag{39}$$

$$\Gamma(t \to qW_{L/R}) = \frac{g_2^2|\vec{q}|}{32\pi}\left\{\left[|L_1^q|^2+|R_1^q|^2\right]\left(1-x_W^2+x_q^2\right)-4x_q\mathrm{Re}\left\{L_1^qR_1^{q*}\right\}\right.$$
$$+\frac{m_t^2}{M_W^2}\left[|L_2^q|^2+|R_2^q|^2\right]\left(1-x_W^2-2x_q^2-x_W^2x_q^2+x_q^4\right)-4x_q\mathrm{Re}\left\{L_2^qR_2^{q*}\right\}$$
$$-2\frac{m_t}{M_W}\mathrm{Re}\left\{L_1^qR_2^{q*}+L_2^qR_1^{q*}\right\}\left(1-x_W^2-x_q^2\right)$$
$$\left.+2\frac{m_t}{M_W}x_q\mathrm{Re}\left\{L_1^qL_2^{q*}+R_2^qR_1^{q*}\right\}\left(1+x_W^2-x_q^2\right)\right\}$$
$$\pm\frac{g_2^2m_t}{64\pi}\frac{m_t^2}{M_W^2}\left\{-x_W^2\left[|L_1^q|^2-|R_1^q|^2\right]+\left[|L_2^q|^2+|R_2^q|^2\right]\left(1-x_q^2\right)\right.$$
$$\left.+2x_W\mathrm{Re}\left\{L_1^qR_2^{q*}-L_2^qR_1^{q*}\right\}+2x_Wx_q\mathrm{Re}\left\{L_1^qL_2^{q*}-R_2^qR_1^{q*})\right\}\right\}$$
$$\times\left(1-2x_W^2-2x_q^2+x_W^4-2x_W^2x_q^2+x_q^4\right) \tag{40}$$

where the upper sign holds for left handed and the lower sign for right handed W-bosons. Furthermore, $x_q \equiv m_q/m_t$, $x_W \equiv M_W/m_t$,

$$|\vec{p}| = \frac{m_t}{2}\sqrt{\lambda(1,x_q^2,x_W^2)}\ , \tag{41}$$

and the Källén function $\lambda(a,b,c) \equiv (a-b-c)^2-4bc$.

The total rate $\Gamma(t \to qW)$ is given by the sum

$$\Gamma(t \to qW) = \Gamma(t \to qW_0)+\Gamma(t \to qW_L) +\Gamma(t \to qW_R)\ , \tag{42}$$

and the corresponding observables are the helicity fractions $F_0 = \Gamma(t \to qW_0)/\Gamma(t \to qW)$, $F_L = \Gamma(t \to qW_L)/\Gamma(t \to qW)$ and $F_R = \Gamma(t \to qW_R)/\Gamma(t \to qW)$. Using the condition $F_0+F_R+F_L \equiv 1$ we get for the normalised differential decay rate [23, 24],

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{3}{8}\left(1-\cos\theta^*\right)^2F_L+\frac{3}{8}\left(1+\cos\theta^*\right)^2F_R +\frac{3}{4}\sin^2\theta^*F_0\ , \tag{43}$$

with the helicity angle θ^*, defined as the angle between the charged lepton three-momentum in the W-boson rest frame and the W-boson momentum in the top-quark rest frame.

The latest experimental measurements from ATLAS [25] and CMS [26] are shown in Tab. I.

For a quantiative analysis we adopt the scheme described above. This means in particular, that we take the relations (34) as equalities, and that we analyse the data in terms of the single quantity

Table I. Helicity fractions @95% CL from ATLAS and CMS for $t \to bW$ decay (see text for references). The errors are statistically and systematically, respectively.

Fraction	ATLAS	CMS
F_0	$0.57 \pm 0.07 \pm 0.09$	$0.567 \pm 0.074 \pm 0.047$
F_L	$0.35 \pm 0.04 \pm 0.04$	$0.393 \pm 0.045 \pm 0.029$
F_R	$0.09 \pm 0.04 \pm 0.08$	$0.040 \pm 0.035 \pm 0.044$

$$\delta L_1^q = \alpha\frac{v^2}{\Lambda^2}\ , \tag{44}$$

where α would be unity in a tree-induced scenario, while $\alpha = 1/(16\pi^2)$ in a loop-induced scenario. In Fig. 2 we plot the helicity fraction F_L and F_R for $t \to bW$ as as function of δL_1^3. The Standard model value corresponds to $\delta L_1^q \equiv 0$ up to very small radiative corrections. The colored bands indicate the data shown in Tab. I. From this we infer that the SM value is well compatible with the current data. However, given the current uncertainties, there is still some room for a nonvanishing δL_1^q. It is interesting to note that for both helicity fractions a region around $\delta L_1^q \sim 0.4$ still is allowed, while the other region constrains $|\delta L_1^q| \leq 0.1$.

The parameter δL_1^q still contains the dependence on Λ, the scale of new physics. Assuming a value of $\Lambda \sim 1$ TeV we end up with $v^2/\Lambda^2 \sim 0.1$. This implies for NP scales around 1 TeV that the the couplings in (33) can still be as large as unity, implying that the current sensitivity

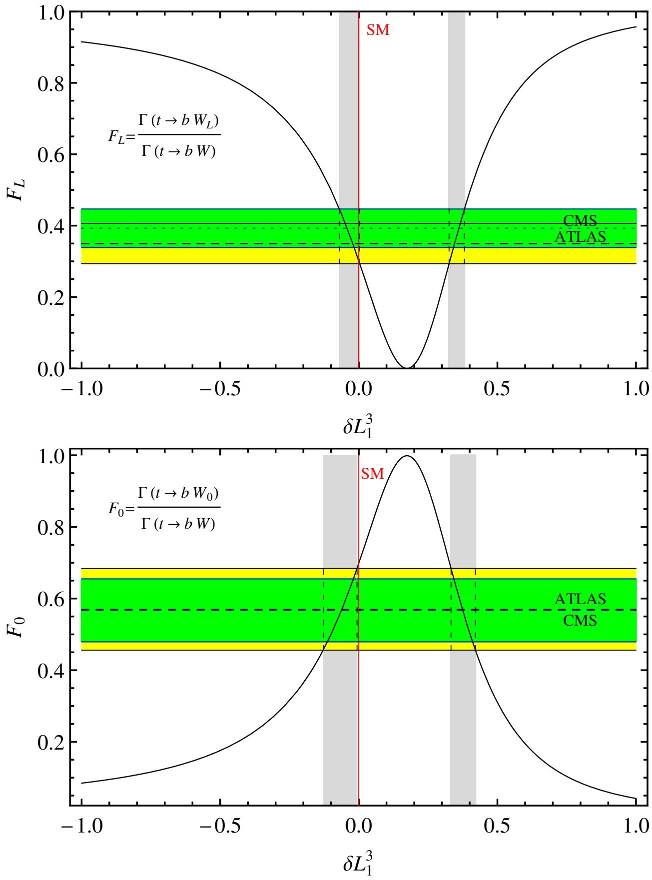

Figure 2. Helicity fractions F_L (left) and F_0 (right) for the decay $t \to bW$ as a function of a possible MFV new physics contribution with the coupling δL_1^q. The horizontal bands indicate the current data from the LHC experiments, the vertical bands indicate the currently allowed range for δL_1^q.

cannot rule out MFV new-physics effects at tree level. In turn, in the loop-induced scenario there is still plenty of room for NP effects.

B. Neutral Currents

The study of Flavour-Changing Neutral Currents (FCNC's) such as $t \to qV$, $q \in \{u, c\}$, $V \in \{Z, \gamma, g\}$ is important in the context of NP analyses, since the contribution of the SM is highly GIM suppressed [27]. A measurement of such a process at the current level of sensitivity would clearly indicate new physics, in particular also implying non-MFV effects; the SM branching ratios ($\mathcal{B}$) are in the region $\mathcal{B}(t \to qZ) \sim \mathcal{O}(10^{-13})$, $\mathcal{B}(t \to q\gamma) \sim \mathcal{O}(10^{-13})$ and $\mathcal{B}(t \to qg) \sim \mathcal{O}(10^{-11})$, [28].

The operators contributing to the FCNC interactions are listed in Tab. II. The experimental signatures of the various channels are quite different, so we study the different processes in the following separately.

Table II. List of operators for $t \to qV$, $q \in \{u, c\}$ and $V \in \{Z, \gamma, g\}$, transitions.

Decay	Operator
$t \to qZ$	$\mathcal{O}_{LL}^{ij(3)}$, $\mathcal{O}_{LL}^{ij(4)}$, $\mathcal{O}_{LL}^{ij(5)}$, $\mathcal{O}_{LL}^{ij(6)}$ $\mathcal{O}_{RR}^{ij(2)}$, $\mathcal{O}_{LR}^{ij(4)}$, $\mathcal{O}_{LR}^{ij(5)}$
$t \to q\gamma$	$\mathcal{O}_{LR}^{ij(4)}$, $\mathcal{O}_{LR}^{ij(5)}$
$t \to qg$	$P_{LR}^{ij(5)}$

1. $t \to qZ$

The effective Lagrangian for the neutral currents involving the Z_0 can be written as

$$\mathcal{L}_{\text{eff}} = \frac{g_2}{\cos\theta_W} Z_\mu \times \Big\{ \bar{q}\gamma^\mu \left(L_1'^q P_L + R_1'^q P_R \right) t - \frac{(i\partial_\nu)}{M_Z} \left[\bar{q} i\sigma^{\mu\nu} \left(L_2'^q P_L + R_2'^q P_R \right) t \right] \Big\} , \tag{45}$$

with the couplings

$$L_1'^q = V_{qb}V_{tb}^* \left[\eta_{LL}^{(3)} \frac{m_b^2}{\Lambda^2} + \eta_{LL}^{(4)} \frac{m_b^2}{\Lambda^2} \tan^2\beta - \frac{\eta_{LL}^{(5)}}{2} \frac{m_b^2}{\Lambda^2} - \frac{\eta_{LL}^{(6)}}{2} \frac{m_b^2}{\Lambda^2} \tan^2\beta \right] , \tag{46}$$

$$R_1'^q = V_{qb}V_{tb}^* \alpha_{RR}^{(2)} \frac{m_b^2}{\Lambda^2} \frac{m_q m_t}{v^2} \frac{1}{\sin^2\beta} , \tag{47}$$

$$L_2'^q = 2V_{qb}V_{tb}^* \frac{m_q}{v} \frac{1}{\sin^2\beta} \left(\cos\theta_W \alpha_{LR}^{(4)*} \frac{m_b^2}{\Lambda^2} - \sin\theta_W \alpha_{LR}^{(5)*} \frac{m_b^2}{\Lambda^2} \right) , \tag{48}$$

$$R_2'^q = 2V_{qb}V_{tb}^* \frac{m_t}{v} \frac{1}{\sin^2\beta} \left(\cos\theta_W \alpha_{LR}^{(4)} \frac{m_b^2}{\Lambda^2} - \sin\theta_W \alpha_{LR}^{(5)} \frac{m_b^2}{\Lambda^2} \right) , \tag{49}$$

where θ_W is the Weinberg angle.

As an order of magnitude estimate, it is worthwhile to note that MFV leads to a significant GIM-like suppres-

sion of this coupling by a factor m_b^2/Λ^2 which is much smaller than the "natural" value of the coupling v^2/Λ^2. Furthermore, also for the relative sizes of the couplings for the various helicity combinations, we get an MFV prediction $\tan\beta \sim 1$ or larger

$$\begin{aligned} R_1^{\prime q} &\sim L_1^{\prime q} \frac{m_q m_t}{v^2} \frac{1}{\tan^2\beta} \ , \\ L_2^{\prime q} &\sim L_1^{\prime q} \frac{m_q}{v} \frac{1}{\tan^2\beta} \ , \\ R_2^{\prime q} &\sim L_1^{\prime q} \frac{m_t}{v} \frac{1}{\tan^2\beta} \ , \end{aligned} \tag{50}$$

Finally we note that there is also a loop-induced contribution from the SM which has been calculated in Ref. [29]. However, the relevant vertex cannot be expressed as a local operator, and hence the expressions are quite cumbersome. On the other hand, since the SM is by construction MFV, the same suppression factors as for the new physics contribution will appear, and since the SM rates are tiny compared to the current experimental limit we take for the SM contribution the simple estimates

$$\begin{aligned} L_{1\mathrm{SM}}^{\prime q} &= V_{qb} V_{tb}^* \frac{1}{16\pi^2} \frac{m_b^2}{v^2} \ , \qquad (51) \\ R_{1\mathrm{SM}}^{\prime q} &= V_{qb} V_{tb}^* \frac{1}{16\pi^2} \frac{m_b^2}{v^2} \frac{m_q m_t}{v^2} \ , \\ L_{2\mathrm{SM}}^{\prime q} &= R_{2\mathrm{SM}}^{\prime q} = V_{qb} V_{tb}^* \frac{1}{16\pi^2} \frac{m_b^2}{v^2} \frac{m_t}{v} \ , \end{aligned}$$

which is numerically very close to the real calculation, i.e. it yields a branching fraction of the order

$$\mathcal{B}_{\mathrm{SM}}(t \to cZ) \sim \left| \frac{1}{16\pi^2} V_{cb} \frac{m_b^2}{v^2} \right|^2 \sim 6 \times 10^{-15} \ . \tag{52}$$

In order to perform a numerical analysis we proceed similarly to the case of charged currents. We take the approximate relations (50) as exact equations and express everything in terms of the new-physics coupling $L_1^{\prime q}$. Including the standard model estimate on the basis of the naive estimate (51) we show in Fig. 3 the branching fraction for $t \to qZ$, $q \in \{u, c\}$.

We note that the natural size of $L_1^{\prime q}$ is in MFV given by $V_{qb} V_{tb}^* m_b^2/\Lambda^2$ assuming tree level FCNC effects. This is for $\Lambda \sim 1$ TeV about 10^{-6} which is one order of magnitude above the SM value. Loop induced new physics effects would show up in MFV for $\Lambda \sim 1$ TeV only at a level of $L_1^{\prime q} \sim 10^{-9}$ which is far below the SM value. In turn, the current experimental limit implies $L_1^{\prime q} \leq 0.01$ which is far above the prediction of any MFV scenario.

2. $t \to q\gamma$ and $t \to qg$

The possible couplings for photonic transitions are more restricted due to electromagnetic gauge invariance.

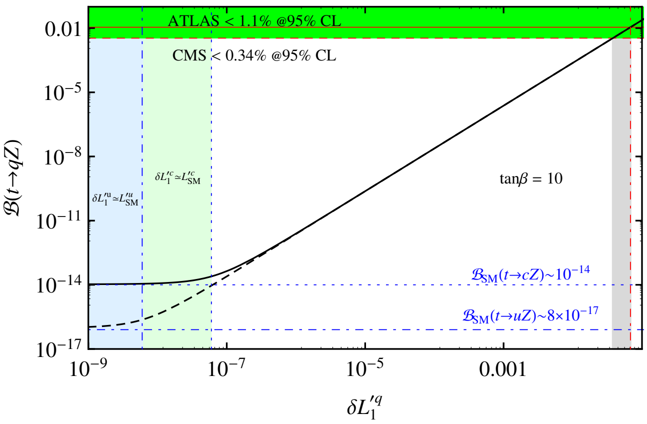

Figure 3. The branching fraction for $t \to Zq$ as a function of the coupling $L_1^{\prime q}$ and $\tan\beta = 10$. The horizontal lines indicate the expectation within the SM and the current limits imposed by ATLAS [30, 31] and CMS [32].

Extracting the relevant terms form the dimension-6 operators we find for the effective Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = -e\mathcal{A}_\mu \frac{(i\partial_\nu)}{m_t} \left[\bar{q} i \sigma^{\mu\nu} \left(L_q^{(2)} P_L + R_q^{(2)} P_R \right) t \right] , \tag{53}$$

where e and $\mathcal{A}_\mu$ are the electron charge and the electromagnetic field, respectively.

For the γtq vertex we get only two independent anomalous coupling constants,

$$\begin{aligned} L_q^{(2)} &= \frac{4 V_{qb} V_{tb}^*}{e} \left(\sin\theta_W \mathcal{C}_{LR}^{32(4)*} + \cos\theta_W \mathcal{C}_{LR}^{32(5)*} \right) \frac{m_b^2}{\Lambda^2} \frac{m_q m_t}{v_1^2} \ , \\ R_q^{(2)} &= \frac{4 V_{qb} V_{tb}^*}{e} \left(\sin\theta_W \mathcal{C}_{LR}^{23(4)} + \cos\theta_W \mathcal{C}_{LR}^{23(5)} \right) \frac{m_b^2}{\Lambda^2} \frac{m_t^2}{v_1^2} \ . \end{aligned} \tag{54}$$

Clearly, in a MFV scenario, the right-handed top-quark yields the dominant contribution,

$$L_q^{(2)} \sim \frac{m_q}{m_t} R_q^{(2)} \ . \tag{55}$$

Taking this as an equation, we can express the rate and the branching fractions in terms of the single coupling $R_q^{(2)}$.

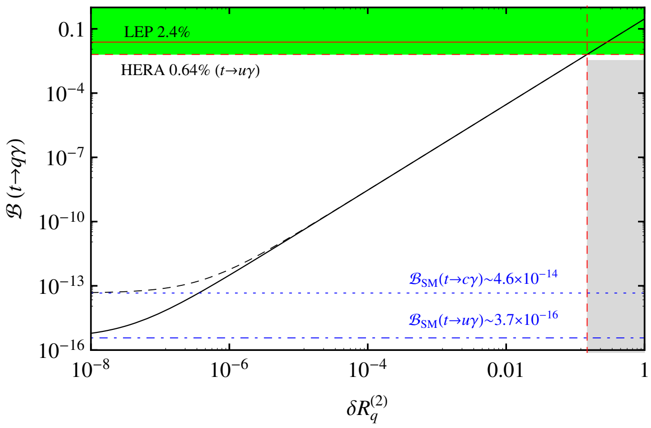

Figure 4. The branching fraction for $t \to q\gamma$ as a function of the coupling $L_1^{\prime q}$. The horizontal lines indicate the expectation within the SM and the current limits imposed by LEP [33–37] and HERA ($t \to u\gamma$) [38], and also from Tevatron: 3.2% [39] which is not shown.

The standard model value for this process is very small. This can be inferred already from a simple order-of-magnitude estimate by just collecting the factors for the loop suppression and the CKM and mass factors. One obtains

$$\mathcal{B}_{\rm SM}(t \to q\gamma) \sim \left| \frac{\alpha}{\pi} V_{tb} V_{qb}^* \frac{m_b^2}{M_W^2} \frac{m_t^2}{v^2} \right|^2 \sim \begin{cases} 4 \times 10^{-14} & \text{for} \quad t \to c\gamma \\ 4 \times 10^{-16} & \text{for} \quad t \to u\gamma \end{cases} , \quad (56)$$

which is numerically close to the values obtained from the full calculation.

As in the case of $t \to Zq$, the MFV expectation is very small. Assuming a tree-like scenario, $R_q^{(2)}$ is of the order $V_{qb} V_{tb}^* m_b^2/\Lambda^2$. For a new physics scale $\Lambda \sim 1$ TeV we end up with a typical expectation $R_c^{(2)} \sim 10^{-7}$ and $R_u^{(2)} \sim 10^{-8}$ for the loop-induced scenario this is even smaller by a factor $1/(16\pi^2)$. Thus, if nature is minimally flavour violating, but otherwise the couplings have natural sizes, the current limits are several orders of magnitude above the expectations.

The case $t \to gq$ is very similar to $t \to q\gamma$; the only difference is the larger strong coupling and larger QCD renormalization effects. The one-loop QCD renormalization is given in the appendix A; although the effects can be sizable, they are still far away from being relevant at the current experimental situation.

Due to QCD gauge invariance the effective interaction has a similar form as the photonic operator

$$\mathcal{L}_{\rm eff} = -g_s G_\mu^a \frac{1}{m_t} \times (-i\partial_\nu) \left[\bar{q} T^a i\sigma^{\mu\nu} \left(\tilde{L}_q^{(2)} P_L + \tilde{R}_q^{(2)} P_R \right) t \right] , (57)$$

with

$$\tilde{L}_q^{(2)} = V_{qb} V_{tb}^* \frac{4}{g_s} K_{LR}^{32(5)*} \frac{m_b^2}{\Lambda^2} \frac{m_q m_t}{v_1^2} , \quad (58)$$

$$\tilde{R}_q^{(2)} = V_{qb} V_{tb}^* \frac{4}{g_s} K_{LR}^{23(5)} \frac{m_b^2}{\Lambda^2} \frac{m_t^2}{v_1^2} , \quad (59)$$

where G_μ^a is the field strength tensor of the gluon fields and T^a are the Gell-Mann matrices.

For $K_{LR}^{32(5)}, K_{LR}^{23(5)} \sim 1$ we get the same order-of-magnitude relation for the couplings as for the photonic case

$$\tilde{L}_q^{(2)} \sim \frac{m_q}{m_t} \tilde{R}_q^{(2)} . \quad (60)$$

and we shall again use this as an equality to perform an MFV analysis in term of the single variable. Fig. 5 shows the current status.

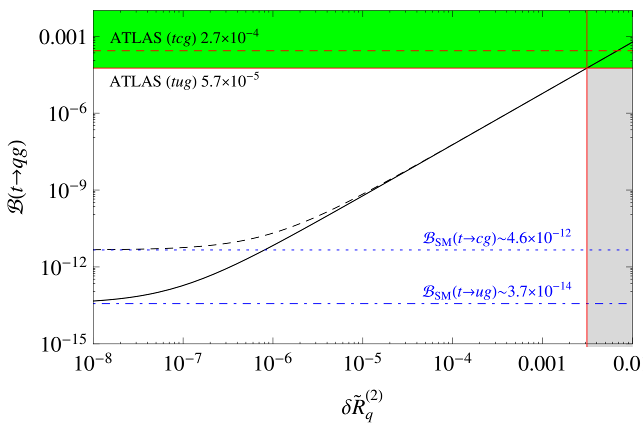

Figure 5. The branching fraction for $t \to qg$ as a function of the coupling $\tilde{R}_q^{(2)}$. The horizontal lines indicate the expectation within the SM and the current limits imposed by ATLAS [40].

The estimate in the standard model is the same as for $t \to q\gamma$ with the electromagnetic coupling replaced by the strong one

$$\mathcal{B}_{\rm SM}(t \to qg) \sim \left| \frac{\alpha_s}{\pi} V_{tb} V_{qb}^* \frac{m_b^2}{M_W^2} \frac{m_t^2}{v^2} \right|^2 \sim \begin{cases} 6 \times 10^{-12} & \text{for} \quad t \to cg \\ 4 \times 10^{-14} & \text{for} \quad t \to ug \end{cases} , \quad (61)$$

which again close to the result fo the full calculation. Concerning the expectations of the MFV scenario, one arrives at the same conclusions as for $t \to q\gamma$: The MFV expectations are still several orders of magnitude away from the experimental sensitivity.

V. CONCLUSIONS

To summarise, we have discussed anomalous, flavour-changing top decays in a model independent way by employing an effective field theory approach. The new element in our analysis is the implementation of minimal flavour violation by setting up a simple scheme with few parameters, which obeys the MFV hierarchical structure. We have calculated the decay rates for the charged current $t \to qW$, $q \in \{d, s, b\}$, as well as for the FCNC couplings tqV, $q \in \{u, c\}$ and $V \in \{Z, \gamma, g\}$, in terms of the effective couplings for different helicities under the assumption that the top-quark is produced in a high energy collision as a quasi-free particle. Comparing such a scenario with the present data shows that there is still plenty of room for NP effects in the anomalous, flavour-changing top couplings, in particular for the flavour-changing neutral current decays.

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Appendix A: QCD Renormalization

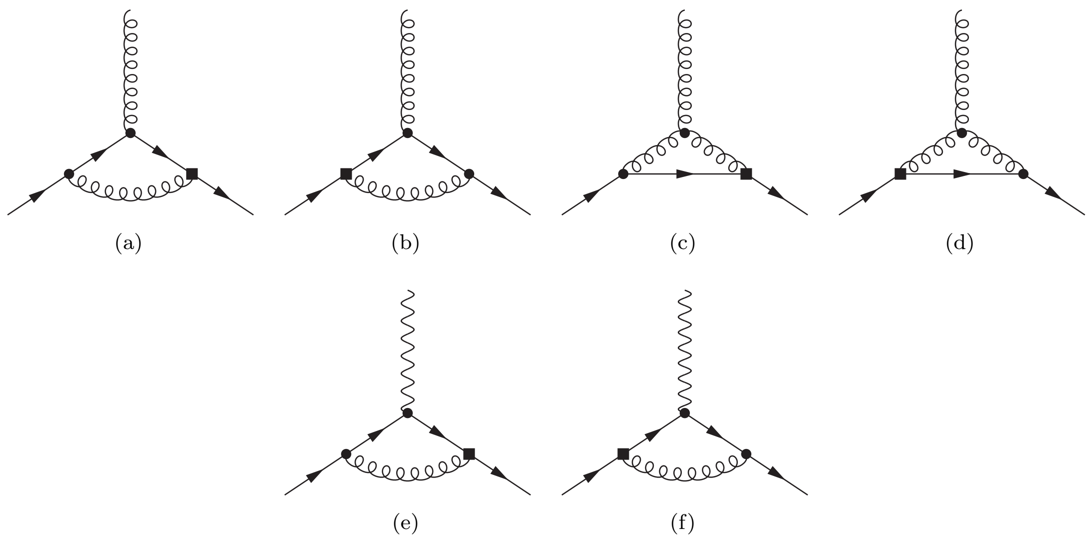

Figure 6. Feynman diagrams for the calculation of the anomalous dimension for the strong decays. Diagrams (e) and (f) including a mixing of strong and electroweak interactions.

The calculation of the anomalous dimension for the $t \to qV$ transitions proceeds along the same lines as we discussed in Sec. IV A for the charged current. However the QCD-like structure of the generalised gtq-vertex, Eq. (57), additional diagrams have to be taken into account. All Feynman diagrams shown in Fig. 6, where Figs. 6(f) and 6(f) represents the mixing of strong and electroweak operators, have to be calculated. We define

$$\vec{\mathcal{C}} = \begin{pmatrix} \tilde{L}_q^{(2)} \\ \tilde{R}_q^{(2)} \\ L_q^{(2)} \\ R_q^{(2)} \\ L_2^{\prime q} \\ R_2^{\prime q} \end{pmatrix} , \tag{A1}$$

$$\vec{\mathcal{O}} = (-i\partial_\nu)\left(\bar{q}\sigma^{\mu\nu}\begin{pmatrix}\frac{1}{m_t}T_a P_L G^a_\mu \\ \frac{1}{m_t}T_a P_R G^a_\mu \\ \frac{1}{m_t}P_L\mathcal{A}_\mu \\ \frac{1}{m_t}P_R\mathcal{A}_\mu \\ \frac{1}{M_Z}P_L Z_\mu \\ \frac{1}{M_z}P_R Z_\mu\end{pmatrix} t\right) , \tag{A2}$$

and get for the 6×6 anomalous dimension matrix for the Wilson coefficients by using Feynman gauge

$$\gamma^T(\mu) = \frac{2\alpha_s(\mu)}{3\pi}\begin{pmatrix} -\frac{215}{32} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{215}{32} & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 & 0 & 0 \\ \cos(2\theta_W)-\frac{1}{4} & 0 & 0 & 0 & 1 & 0 \\ 0 & \cos(2\theta_W)-\frac{1}{4} & 0 & 0 & 0 & 1\end{pmatrix} . \tag{A3}$$

This matrix describes the mixing of QCD-like operators and neutral weak electroweak operators and determines the running of the Wilson coefficients.

The solution of the renormalization group equation (RGE),

$$\frac{d\vec{\mathcal{C}}}{d\ln\mu} = \gamma^T(\mu)\cdot\vec{\mathcal{C}} , \tag{A4}$$

is straightforward, and we find

$$\tilde{I}^{(2)}_q(\mu) = \tilde{I}^{(2)}_q(\Lambda)\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}\right)^{-\frac{215}{24\beta_0}} , \tag{A5}$$

$$I^{(2)}_q(\mu) = \left[3\tilde{I}^{(2)}_q(\Lambda) + I^{(2)}_q(\Lambda)\right]\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}\right)^{\frac{4}{3\beta_0}} - 3\tilde{I}^{(2)}_q(\Lambda)\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}\right)^{-\frac{215}{24\beta_0}} , \tag{A6}$$

$$\begin{aligned} I'^q_2(\mu) = &\left[\left(\cos(2\theta_W)-\frac{1}{4}\right)\tilde{I}^{(2)}_q(\Lambda) + I'^q_2(\Lambda)\right]\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}\right)^{\frac{4}{3\beta_0}} \\ &-\left(\cos(2\theta_W)-\frac{1}{4}\right)\tilde{I}^{(2)}_q(\Lambda)\left(\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}\right)^{-\frac{215}{24\beta_0}} , \end{aligned} \tag{A7}$$

with $I\in\{L,R\}$. The scale dependencies of L'^q_1 and R'^q_1 are given by

$$L'^q_1(\mu) = L'^q_1(\Lambda) ,\; R'^q_1(\mu) = R'^q_1(\Lambda) . \tag{A8}$$

With these relations it is not difficult to check the logarithmic terms of the one-loop calculation including the logarithmic corrections to every order in $\alpha_s \ln\Lambda/\mu$, $\mu\sim m_t$.

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